**【1】Closed immersion**

Let $f : Z \to X$ be a morphism of schemes. Then $f$ is a closed immersion

$\Longleftrightarrow \exists X = \bigcup U_j$ (affine open covering) with $U_j = \mathrm{Spec}(R_j)$ $\exists I_j \subset R_j$ (ideal) s.t.

$i^{-1}(U_j) = \mathrm{Spec}(R_j/I_j)$ as schemes over $U_j$.

**【2】Reduced induced closed subscheme**

Let $X$ be a scheme, $Z \subset X$ be a closed subscheme and $Y$ be a reduced scheme.

A morphism $f : Y \to X$ factors through $Z \Longleftrightarrow f(Y) \subset Z$(set theoretically).

**【3】Irreducible**

Let $X$ be a scheme. $X$ is irreducible

$\Longleftrightarrow \exists X = \bigcup U_j$ (affine open covering) s.t. $U_j$ is irreducible and $U_i \cap U_j \neq \emptyset$ $(\forall i \neq j)$

**【4】[*1]Irreducible components and Connected components**

Let $X$ be a scheme Then (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii):

(i) All connected components are open.

(ii) $X$ is the disjoint union of irreducible components.

(iii) $\forall\, x \in X,\ \mathcal{O}_{X,x}$ is a domain.

Moreover, if $X$ is Noether, (i), (ii), (iii) are equivalent.

**【5】(locally) Noether and (locally) of finite type**

Let $f : X \to S$ be a morphism (locally) of finite type and $S$ be (locally) Noetherian.

Then $X$ is (locally) Noetherian.

**【6】affine morphism**

Let $f : X \to S$ be a morphism. $f$ is an affine morphism (i.e. the inverse image of any affine open subset of $S$ is affine)

$\Longleftrightarrow \exists S = \bigcup U_j$ (affine open covering) s.t. $f^{-1}(U_j)$ is affine.

**【7】integral (resp. finite) morphism**

Let $f : X \to S$ be a morphism. $f$ is an integral (resp. finite)morphism

$\Longleftrightarrow \exists S = \bigcup U_j$ (affine open covering) s.t. $f^{-1}(U_j)$ is affine and $\mathcal{O}_S(U_j) \to \mathcal{O}_X(f^{-1}(U_j))$ is integral (resp. finite).

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[*1] prf is written in [note,Liu,24]