

問題 8.1. 絶対値 ① $\alpha = 1 - 2i, \beta = 3 - i$ とする.(1) $|\alpha|, |\alpha\beta|, \left|\frac{\alpha}{\beta}\right|, |\bar{\alpha}\beta^2|$ を求めよ.(2) α と β の距離を求めよ.(3) $\left|\alpha + \frac{1}{\alpha}\right|$ を求めよ.

$$(1) \quad |\alpha| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

$$|\beta| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$$|\alpha\beta| = |\alpha| \cdot |\beta| = 5\sqrt{2}$$

$$\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|} = \frac{1}{\sqrt{2}}$$

$$|\bar{\alpha}\beta^2| = |\bar{\alpha}| \cdot |\beta|^2 = |\alpha| \cdot |\beta|^2 \star 1 \\ = 10\sqrt{5}$$

$$(2) \quad |\alpha - \beta| = |2 + i| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$(3) \quad \left|\alpha + \frac{1}{\alpha}\right|^2 = \left(\alpha + \frac{1}{\alpha}\right) \cdot \overline{\left(\alpha + \frac{1}{\alpha}\right)}$$

$$= \left(\alpha + \frac{1}{\alpha}\right) \left(\bar{\alpha} + \frac{1}{\bar{\alpha}}\right)$$

$$= \alpha\bar{\alpha} + 2 + \frac{1}{\alpha\bar{\alpha}} = |\alpha|^2 + 2 + \frac{1}{|\alpha|^2}$$

$$= 5 + 2 + \frac{1}{5} = \frac{36}{5}$$

$$\therefore \left|\alpha + \frac{1}{\alpha}\right| = \frac{6}{\sqrt{5}}$$

Point $\star 1 \quad |\bar{\alpha}| = |\alpha|$ に注意。

問題 8.2. 絶対値②

α, β を複素数とする. $|\alpha| = 1, |\beta| = 2, |\alpha + \beta| = 2$ のとき, $|\alpha - \beta|$ を求めよ.

$$|\alpha + \beta| = 2 \text{ より}$$

$$(\alpha + \beta)(\overline{\alpha + \beta}) = 2^2$$

$$(\alpha + \beta)(\overline{\alpha} + \overline{\beta}) = 4$$

$$\alpha\overline{\alpha} + \alpha\overline{\beta} + \overline{\alpha}\beta + \beta\overline{\beta} = 4$$

$$\alpha\overline{\beta} + \overline{\alpha}\beta = 4 - 1 - 4 = -1.$$

$$\begin{aligned} \therefore |\alpha - \beta|^2 &= (\alpha - \beta)(\overline{\alpha} - \overline{\beta}) \\ &= |\alpha|^2 + |\beta|^2 - (\alpha\overline{\beta} + \overline{\alpha}\beta) \\ &= 1 + 4 - (-1) = 6. \end{aligned}$$

$$\therefore |\alpha - \beta| = \sqrt{6}$$