問題 9. 極形式

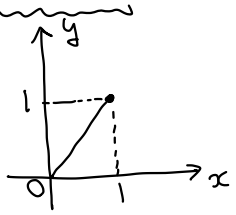
次の複素数を極形式で表せ.

(1) $1+i$　(2) $\sqrt{2}-\sqrt{6}i$　(3) -2　(4) i　(5) $\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}$

(6) $-2(\cos\theta-i\sin\theta)$　(θ は実数)　(7) $1+\cos\theta+i\sin\theta$　$(0<\theta<\pi)$

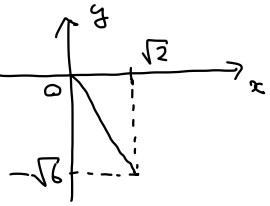
(1) $1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)$

$=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$



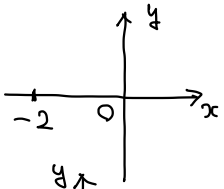
(2) $\sqrt{2}-\sqrt{6}i=2\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}i\right)$

$=2\sqrt{2}\left(\cos\frac{5}{3}\pi+i\sin\frac{5}{3}\pi\right)$

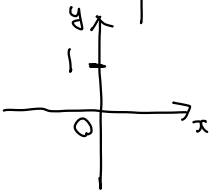


(3) -2

$=2(\cos\pi+i\sin\pi)$.

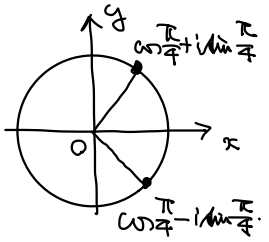


(4) $i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$.



(5) $\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}$

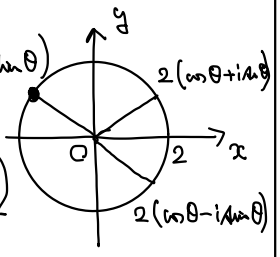
$=\cos\frac{7}{4}\pi+i\sin\frac{7}{4}\pi$.



(6)

$-2(\cos\theta-i\sin\theta)$

$=2(\cos(\pi-\theta)+i\sin(\pi-\theta))$



(7) $1+\cos\theta+i\sin\theta$.

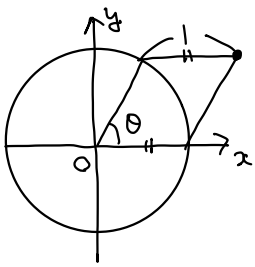
絶対値は,

$\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}$

$=\sqrt{2+2\cos\theta}$

$=2\sqrt{\frac{1+\cos\theta}{2}}=2\sqrt{\cos^2\frac{\theta}{2}}=2\cos\frac{\theta}{2}$

$\left(0<\frac{\theta}{2}<\frac{\pi}{2}\right)$



よって,

$1+\cos\theta+i\sin\theta$

$=2\cos\frac{\theta}{2}\left(\frac{1+\cos\theta}{2\cos\frac{\theta}{2}}+i\frac{\sin\theta}{2\cos\frac{\theta}{2}}\right)$

$=2\cos\frac{\theta}{2}\left(\frac{\cos^2\frac{\theta}{2}}{\cos\frac{\theta}{2}}+i\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}}\right)$

$=2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)$